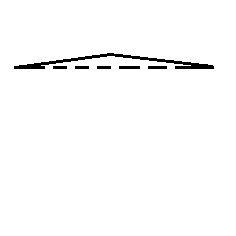
**Homework 7 Solutions: due when the universe ends**

**Problem 1.** Musical aside: The notes that comprise the bottom octave on a piano are:

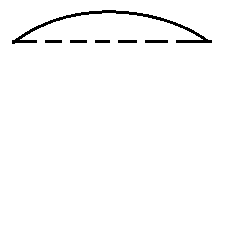
|  |  |
| --- | --- |
| **Note** | **Frequency (Hz)** |
| C1 | 32.7 |
| D1 | 36.7 |
| E1 | 41.2 |
| F1 | 43.7 |
| G1 | 49.0 |
| A1 | 55.0 |
| B1 | 61.7 |

Each successive octave doubles the frequencies. For instance Middle C, i.e. C4, has a frequency of C1∙23 = 262Hz. Now that that’s settled….

When you pluck a stringed instrument, you give it an initial shape D(x,0) that is somewhat triangular-looking. A triangular shape is not a sinusoidal shape, and so it will not be characterized by a *single* wavelength (and frequency) of oscillation like sinusoids are. Rather, the wavelength (and frequency) of oscillation will be comprised of certain percentages of the resonant waveforms’ wavelengths (and frequencies) of oscillation. The resonant waveform which will contribute most is the one that most closely resembles D(x,0). Since the initial shape of a plucked string looks like this:



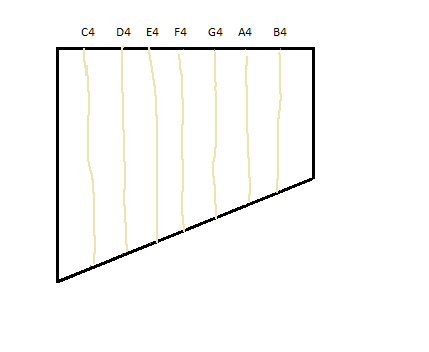
And the resonant waveform which most closely matches this shape is the ‘fundamental’, or ‘first’, one:



So the wavelength (frequency) that you hear the most when you pluck a string is the fundamental wavelength (frequency), though you will hear the higher resonances to a lesser extent, nonetheless.

(a) Now suppose you wanted to make a stringed instrument, like a harp. Supposing each string had a mass density μ = 0.010kg/m, and were tightened to a tension T = 400N, what ought to be the lengths of the seven strings so that their fundamental frequencies would be the following notes. Be sure to do the whole derivation starting from Δφ = 2πm, m1/2. ☺

|  |  |
| --- | --- |
| **Note** | **Length (cm)** |
| C4 |  |
| D4 |  |
| E4 |  |
| F4 |  |
| G4 |  |
| A4 |  |
| B4 |  |



Resonant wavelengths are:



And so the resonant frequencies are:



And so the fundamental frequency is:



And so the requisite lengths are:



So,

|  |  |  |
| --- | --- | --- |
| **Note** | **Frequency (Hz)** | **Length (cm)** |
| C4 | C1∙8 = 262 | 38.2 |
| D4 | D1∙8 = 294 | 34.0 |
| E4 | E1∙8 = 330 | 30.3 |
| F4 | F1∙8 = 392 | 28.7 |
| G4 | G1∙8 = 440 | 25.5 |
| A4 | A1∙8 = 494 | 22.7 |
| B4 | B1∙8 = 524 | 20.2 |

(b) Which harmonics of the string, if any, would correspond to the next two octaves? And how could these notes be played?

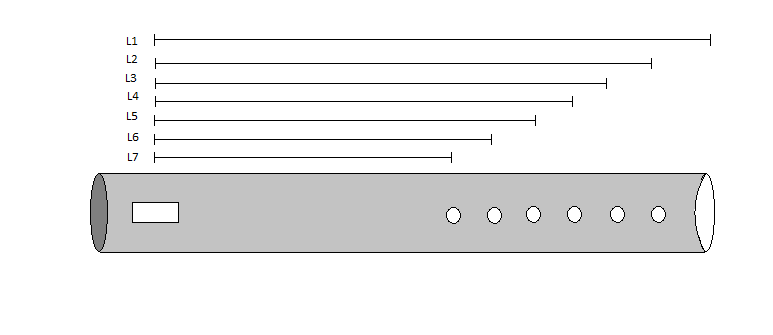
The second, third, and fourth harmonics are:



So the second corresponds to the next octave, and the fourth corresponds to the octave after that. You could play these by plucking the string in a manner similar to these harmonics: a figure eight for the second harmonic, and a figure ‘16’ for the fourth. Or better, just pluck half the the string for the second, and a quarter of the string for the fourth.

**Problem 2.** When you play a wind or reed instrument, like a flute, or clarinet you do not control the shape of the disturbance you create in the body of the instrument, like you do with a string instrument. Rather, by blowing over the opening (‘slow’, or ‘fast’, etc.) you control the *frequency(ies)* of disturbance you create in the instrument’s air column. The faster you blow (overblowing it’s called), the higher the frequency you may produce. Blowing over the opening will create waves with a *range* of frequencies actually; only the ones whose frequencies match a resonant waveform will be amplified.

(a) Now suppose you wanted to make a flute. To create the notes in the octave, we need to have seven different lengths of air columns. A shortcut is to simply drill 6 holes. By covering up all the holes, we have an open ended column of air L1 long. If we cover up all but the last, then we have an open-ended column L2 long, and so on. So what should these L’s be to create notes C4 through B4 again? Be sure to do the whole derivation starting from Δφ = 2πm, m1/2. Take the air to have molar mass mmol = 0.029kg, to be diatomic (γ = 1.4), and to be at temperature T = 310K.

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The resonant wavelengths are:



And frequencies,



The fundamental frequencies are:



Solving for length we have:



Filling in the same frequencies as before, we get:

|  |  |
| --- | --- |
| **Note** | **Length (cm)** |
| C4 | 67.2 |
| D4 | 59.9 |
| E4 | 53.3 |
| F4 | 50.4 |
| G4 | 44.9 |
| A4 | 40.0 |
| B4 | 35.6 |

(b) Suppose the room were colder. What would you have to do to these lengths to compensate (this is why you can adjust the length of wind instrument)?

If T is lower, then v is lower. And we can see from L = 176/f, that we’d have to shorten L.

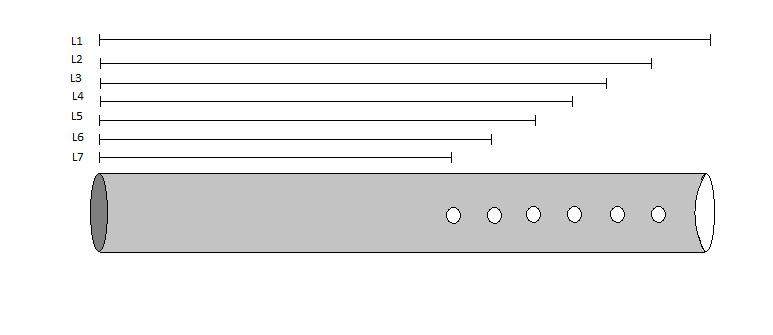
(c) Which harmonics of the flute, if any, would correspond to the next two octaves? And how could these notes be played in principle (kind of already said it)?

The second, third, and fourth harmonics are:



So the second corresponds to the next octave, and the fourth corresponds to the octave after that. You could play these by overblowing, for instance.

**Problem 3.** Suppose you wanted to make a recorder (like a clarinet), which differs from a flute in that the closed end over which you blow makes that end of the instrument a ‘hard’ or ‘closed’ end. We make a similar construction. So what should these L’s be to create notes C4 through B4 again? Be sure to do the whole derivation starting from Δφ = 2πm, m1/2, s’il vous plait. And take the air to have molar mass mmol = 0.029kg, to be diatomic (γ = 1.4), and to be at temperature T = 310K. Note the mouthpiece isn’t really shown, because I’m using MS Paint, and yeah.

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The resonant wavelengths are:



And frequencies,



The fundamental frequencies are:



Solving for length we have:



Filling in the same frequencies as before, we get:

|  |  |
| --- | --- |
| **Note** | **Length (cm)** |
| C4 | 33.6 |
| D4 | 30.0 |
| E4 | 26.6 |
| F4 | 25.2 |
| G4 | 22.4 |
| A4 | 20.0 |
| B4 | 17.8 |

(b) Suppose the room were warmer. What would you have to do to these lengths to compensate (this is why you can adjust the length of such instruments too)?

If T is higher, then v is higher. And we can see from L = 88/f, that we’d have to lengthen L.

(c) Which harmonics of the recorder, if any, would correspond to the next two octaves?

The second, third, and fourth harmonics are:

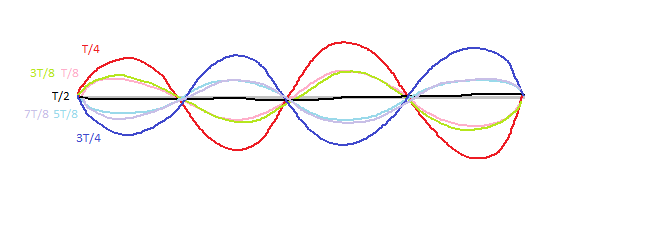


So actually, none of the higher harmonics corresponds to the octave up, or any of the higher octaves. So these higher notes couldn’t *simply* be played by overblowing. You’d have to do that in combination with pressing some more keys (closing holes, etc.).

**Problem 4.** We gonna draw some pictures! For each of the following three cases, and for the 4th resonant waveform, specify its speed, wavelength, frequency, and period. And then draw what it looks like in seven 1/8 period incrememts (should return to its initial state by the eighth increment), starting from the equilibrium position at t = 0.

(a) A harp string with length ℓ = 75cm, mass m = 0.007kg, under tension T = 600N.

Looks like,



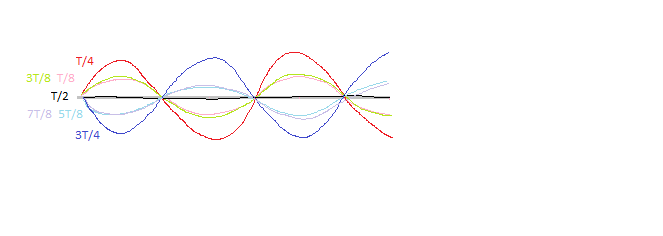


And:



(b) A trumpet with tube length ℓ = 50cm (all valves closed), enclosing air (γ = 1.4, molar mass m = 0.029kg, gas constant R = 8.31 J/mol∙K) at temperature T = 310K.

Looks like,



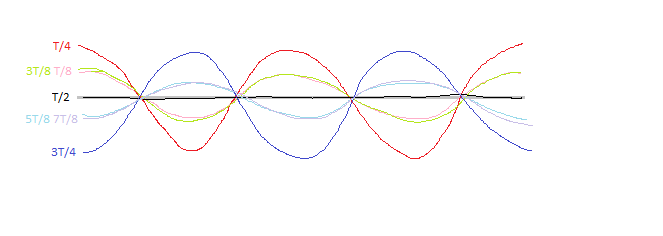


And,



(c) A flute of length ℓ = 30cm (all valves closed), enclosing air (γ = 1.4, molar mass m = 0.029kg, gas constant R = 8.31 J/mol∙K) at temperature T = 310K.

Looks like,





And,



**Problem 5.** The resonant waveleforms analyzed above were those of the string or of the air *inside* the instrument. But you don’t hear the string itself or the air inside the instrument itself. The soundwaves you hear are those which these resonant waveforms setup in the air *surrounding* the instrument. Suppose the surrounding air temperature is T = 280K (same γ, molar mass, R). In each of the cases above, give the velocity, wavelength, frequency, and period of the wave *you* hear.

(a) Easy peasy. We analyze this more or less like a wave transmitting into another medium. Speed and wavelength is determined by medium, but frequency (and period) is invariant.



(b) Same as above,

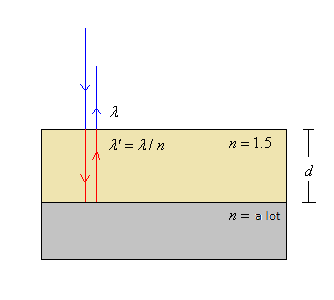


(c) And,



**Problem 6.** Suppose we wanted to make an airplane invisible to a certain radar wavelength, say λ = 3cm. We could do this by coating the metallic surface of the plane with a polymer (for which say n = 1.50) to a certain thickness to ensure that the radar waves that reflect off the plane cancel each other out. Metallic surfaces have, for our purposes, an index of refraction of n = ∞ (they are really good reflectors of radar waves). So what minimum thickness would do the job? Be sure to do the whole derivation starting from Δφ = 2πm, m1/2.

The original wave (λ = 3cm) will be incident on the polymer, and will reflect, undergoing a phase shift. Transmitted wave will also under go a phase shift,



For the waves to cancel, we need:



And so the minimum d would be given by:



**Problem 7.** After having aced all of your exams you treat yourself to a bubble bath.  And you notice light shining through a thin soap bubble membrane with index of refraction n = 1.7 and thickness d = 0.3μm.  (a) What wavelengths of visible lighty would be most strongly transmitted through the membrane? Need to see wavelength derivations ☺. Is that getting old?

Hmmmmm……well the wavelengths that perfectly transmit are those which will destructively interfere upon reflection. When the wave hits the membrane it will reflect off the front surface and the back surface. It will invert off the front surface (call this the first reflected wave), but it won’t reflect off the back surface (call this the second reflected wave). So we have (it’s sooooo easy!):



Now λ2 is the wavelength in the bubble, not in air (and it’s *that* wavelength we need). So we say: n1λ1 = n2λ2 →



Filling in m­1/2 = 0.5, 1.5, 2.5, 3.5, we get λ = 510nm as the only wavelength in visible range.

(b) Which wavelengths would be most strongly reflected?

Now we want constructive interference upon reflection. So only thing that changes is the m:



Now λ2 is the wavelength in the bubble, not in air (and it’s *that* wavelength we need). So we say: n1λ1 = n2λ2 →



Filling in m­1/2 = 0, 1, 2, 3, we get λ = 680nm and 408nm as the wavelengthsin visible range.